The following is a tretise on fan-duct Systems based on the discussion in the CFAST Technical Note 1431. It is intended to summarize the more detailed information available in that document.

Fan-duct systems are commonly used in buildings for heating, ventilation, air conditioning, pressurization, and exhaust. Cross ventilation is occasionally used without heating or cooling. Generally systems that maintain comfort conditions have either one or two fans. Residences often have a systems with a single fan. In this system return air from the living quarters is drawn in at one location, flows through filter, fan and coils, and is distributed back to the residence. This system does not have the capability of providing fresh outside air. These systems are intended for applications where there is sufficient natural air leakage through cracks in walls and around windows and doors for odor control. Further information about these systems is presented in the by Klote and Milke [1] and the American Society of Heating, Refrigerating and Air Conditioning Engineers [2].

The model for mechanical ventilation used in CFAST is based on the theory of networks and is based on the model developed by Klote [3]. This is a simplified form of Kirchoff's law which says that flow into a node must be balanced by flow out of the node. There is a close analog to electrical networks for which the flow consists of electrons. In the case of ventilation, the flow is formed by molecules of air. The conservation equation differs slightly from that of an electrical system, but the basic ideas carry over. For the former case, we have:

$$\text{voltage} = \text{current} \times \text{resistance} .$$

In the present case we have

$$\text{pressure change} = \text{mass flow} \times \text{resistance} .$$

So the application of network theory is used, although the circuit laws are slightly different. In practice, as with the electrical analog, one solves the problem by summing all of the equations for the nodes, and requires that the mass be conserved at each node. Thus we turn the equation around and put it into the form

$$\text{mass flow} = \text{conductance} \times (\text{pressure drop across a resistance})^{1/2} .$$

For each node, this flow must sum to zero. There are several assumptions which are made in computing this flow in ducts, fans, elbow, *etc*. First, we assume unidirectional flow. Given the usual size of ducts, and the nominal presence of fans, this is quite reasonable. Also, the particular implementation used here [3] does not allow for reverse flow in the fans. The difficulty lies in describing how a fan behaves in such a case.

Each fan-duct system is represented as a network of nodes, each at a specific temperature and pressure. The nodes may be connected by fans, ducts, fittings and other components. Except for fans, air flows through these components from nodes of higher pressure to nodes of lower pressure. For example, the residential system illustrated in Figure 2(a) is represented in Figure 2(b) as a network of a fan, eight resistances and ten nodes. These resistances incorporate all the resistance to flow between nodes. For instance, the equivalent resistance, R_1, between nodes 1 and 2 accounts for resistances of the inlet, duct, filter and connection to the fan.

Given that we can describe mass flow in terms of pressure differences and conductance, the conservation equation for each node is

$$\sum_j \dot{m}_{i,j} = 0. \tag{1}$$

The index "j" is a summation over connections to a node, and there is an equation "i" for each node. The remaining problem is to specify the boundary conditions. At each connection to a compartment, the pressure is specified. Then, given that flow at each connection is unidirectional (at a given instant of time, the flow is either all into or all out of a given connection), the mass and enthalpy flow into or out of a room can be calculated explicitly. Thus we end up with a set of equations of the form

$$\begin{aligned} f_1(P_1, P_2, \ldots) &= 0 \\ &\vdots \\ f_i(P_1, P_2, \ldots) &= 0 \\ &\vdots \\ f_n(P_1, P_2, \ldots) &= 0. \end{aligned} \tag{2}$$

This is an algebraic set of equations that is solved simultaneously with the equations for flow in the compartments.

The equations describe the relationship between the pressure drop across a duct, the resistance of a duct, and the mass flow. The pressure can be changed by conditions in a compartment, or a fan in line in the duct system. Resistance arises from the finite size of ducts, roughness on surfaces, bends and joints. To carry the electrical analog a little further, fans act like constant voltage sources. The analogy breaks down, however, in that the voltage, current and resistance are related by the square of the current, rather than being linearly proportional. Since we are using the current form of the conservation equation to balance the system, the flow can be recast in terms of a conductance

$$\dot{m} = G\sqrt{\Delta P}. \tag{3}$$

The conductance can be expressed generally as

$$G = \sqrt{\frac{2\rho}{C_0}}\, A_0 \tag{4}$$

where C_0 is the flow coefficient, and A_0 is the area of the inlet, outlet, duct, contraction or expansion joint, coil, damper, bend, filter, and so on. Their values for the most common of these items are tabulated in the ASHRAE Handbook [4].

The mechanical ventilation system is partitioned into one or more independent systems. Differential equations for species for each of these systems are derived by lumping all ducts in a system into one pseudo tank. This set of equations is then solved at each time step. Previously the mechanical ventilation computations in CFAST were performed as a side calculation using time splitting. This could cause problems since time-splitting methods require that the split phenomenon (the pressures and temperatures in this case) change slowly compared to other phenomenon such as room pressures, layer heights etc. The pressures at each internal node and the temperatures in each branch (duct, fan) are now determined explicitly by the solver, once again using conservation of mass and energy discussed in this section.

Ducts: Ducts are long pipes through which gases can flow. They have been studied much more extensively than other types of connections. For this reason, eq (4) can be put into a form which allows one to characterize the conductance in more detail, depending on the type of duct (e.g., oval, round, or square). The form derives from the Darcy equation and is

$$G = \sqrt{\frac{FL}{2\rho D_e A_0^2}} , \tag{5}$$

where F is the friction factor and can be calculated from
The temperature for each duct d is determined using the following differential equation:

accumulated heat = (heat in - heat out) - convective losses through duct walls

$$c_v \rho_d V_d \frac{dT_d}{dt} = c_p m_d (T_{in} - T_{out}) - h_d A_d (T_d - T_{amb}) \tag{6}$$

where c_v, c_p are the specific heats at constant volume, pressure; V_d is the duct volume, ρ_d is the duct gas density, dT_d/dt is the time rate of change of the duct gas temperature, m_d is the mass flow rate,T_{in} and T_{out} are the gas temperatures going into and out of the duct, c_d, A_d are the convective heat transfer coefficient and surface area for duct d and T_{amb} is the ambient temperature. The first term on the right hand side of eq (6) represents the net gain of energy due to gas transported into or out of the duct. The second term represents heat transferred to the duct walls due to convection. In version 1.6, the loss coefficient is set to zero. We retain the form for future work. The differential and algebraic (DAE) solver used by CFAST solves eq (6) exactly as written. A normal ordinary differential equation solver would require that this equation be solved for dT/dt. By writing it this way, the duct volumes can be zero which is the case for fans.

Fans: There are two general fan classifications: centrifugal and axial. Figure 3 illustrates the basic parts of a centrifugal fan. Flow within a centrifugal fan is primarily in a radial direction to the impeller. Centrifugal fans can produce large static pressures as high as 4000 Pa (16 in H_2O)

with efficiencies typically from 65 to 80%. Flow within an axial fan is parallel to the shaft. Propeller fans are a type of axial fan that are common in many applications including exhaust. Many propeller fans can produce no more than 300 Pa (1.2 in H_2O) with efficiencies of only 25 to 40%.

Fan Performance: This section provides background information about fan performance. For more information about fans, readers are referred to Jorgensen (1983) and ASHRAE (1992). Typical performance of a fan operating at constant impeller speed is illustrated in figure 6. For this figure, Δp_f is the static pressure of the fan, and $\dot{V}_f$ is the volumetric flow of the fan.

Fans operating in the positively sloping portion of a fan curve exhibit unstable behavior called surging or pulsing. Unstable flow consists of violent flow reversals accompanied by significant changes in pressure, power and noise. There is little information about how long a fan can operate in the unstable region before it is destroyed.

Backward flow can be exhibited by all types of fans. The wind blowing into the outlet of a propeller fan can result in backflow, and pressures produced by fires could also produce backflow. As Δp_f becomes negative, the flow increases with decreasing Δp_f until a choking condition develops at point E.

It is common practice in the engineering community and fan industry to represent fan performance with Δp_f on the vertical axis and $\dot{V}_f$ on the horizontal axis. Probably the reason is that $\dot{V}_f$ can be thought of as a single valued function of Δp_f for flow in the first and second quadrants of a fan curve. Fan manufacturers generally supply flow-pressure data for the normal operating range, and they often supply data for the rest of the fan curve in the first quadrant. Specific data is not available for either second or fourth quadrant flow. No approach has been developed for simulation of unstable fan operation, and numerical modeling of unstable flow would be a complicated effort requiring research.

Fan manufacturer data is routinely either in tabular or graphical form. As indicated by Jorgensen [5], the use of a polynomial form of fan curve is common within the industry.

$$\dot{V}_f = B_1 + B_2 \Delta p_f + B_3 (\Delta p_f)^2 + \cdots + B_n (\Delta p_f)^{n-1} \tag{7}$$

The coefficients can be entered as data or calculated by least squares regression from flow and pressure data. For constant volumetric flow applications, the only non-zero coefficient in eq. 8 is B_1 ($n = 1$). For incompressible fluids, eq. 8 is independent of temperature and pressure. For fan data at 20°C, compressibility effects amount to an error of about 6% at a temperature of 200°C.

Effective Resistances: The resistance, R, of a flow element can be defined as

$$R = \frac{\sqrt{\Delta p}}{\dot{m}} \tag{8}$$

where Δp is the pressure loss through the element corresponding to a mass flow rate, $\dot{m}$. The effective resistance between two nodes is always positive, however, sometimes one of the resistances between nodes can be negative as will be explained later. To account for this, $R = K^{1/2}$ can be substituted into eq. 8 to give

$$\Delta p = K\dot{m}^2 \tag{9}$$

The total pressure loss, Δp_t, from one node to the next is the sum of the losses, Δp_i, through each flow element, i, between the nodes.

$$\Delta p_t = \sum_i \Delta p_i \tag{10}$$

The effective value, K_e, relates the total pressure loss to the mass flow rate as $\Delta p_t = K_e \dot{m}^2$, and K_i relates the pressure loss through element i as $\Delta p_i = K_i \dot{m}^2$. These pressure losses can be substituted into eq. 10, and canceling like terms yields

$$K_e = \sum_i K_i \tag{11}$$

Values of K_i can be calculated for each element using equations developed later, and K_e can be calculated by eq. 11.

Resistance of Ducts: For a straight section of duct with constant cross sectional area, the Bernoulli equation incorporating pressure loss, Δp_{fr}, due to friction is commonly written

$$p_1 - p_2 = \Delta p_{fr} + \rho g(Z_1 - Z_2) \tag{12}$$

where the subscripts 1 and 2 refer to the duct inlet and outlet respectively, p is pressure, Z is elevation, g is the acceleration due to gravity, and ρ is the density of the gas. The pressure loss due to friction is expressed by the Darcy equation in most elementary treatments of flow in pipes and ducts [6], [7], [8].

$$\Delta p_{fr} = f\frac{L}{D_e}\frac{\rho U^2}{2} \tag{13}$$

where f is the friction factor, L is the duct length, D_e is the effective diameter of the duct and U is the average velocity in the duct ($\dot{m} = \rho UA$ where A is the cross-sectional area of the duct). For a circular duct, the effective diameter is the duct diameter. For rectangular duct, Huebscher [9] developed the relationship

$$D_e = 1.30 \frac{(ab)^{0.625}}{(a + b)^{0.250}} \tag{14}$$

where a is the length of one side of the duct, and b is the length of the adjacent side. For flat oval duct, Heyt and Diaz [10] developed the relationship

$$D_e = \frac{1.55 A^{0.625}}{P^{0.200}} \tag{15}$$

where A and P are the cross-sectional area and the perimeter of the flat oval duct. The area of a flat oval duct is

$$A = (\pi b^2/4) + b(a - b) \tag{16}$$

and the perimeter of a flat oval duct is

$$P = \pi b + 2(a - b) \tag{17}$$

where a is the major dimension of the flat oval duct, and b is the minor dimension of the duct. Combining eqs. 8 and 13 results in

$$\dot{m}_{ij} = \frac{1}{R}\sqrt{p_j - p_i + \rho g(Z_j - Z_i)} \tag{18}$$

Combining eqs. 10 and 14 results in

$$K = \frac{fL}{2D_e \rho A^2} \tag{19}$$

where A is the cross sectional area of the duct. Colebrook developed the following equation for the friction factor [11].

$$\frac{1}{\sqrt{f}} = -2Log_{10}\left(\frac{\varepsilon}{3.7\,D_e} + \frac{2.51}{R_e\sqrt{f}}\right) \tag{20}$$

where R_e is the Reynolds number (UD_e/υ where υ is the kinematic viscosity) and ε is the roughness of the inside surface of the duct. Data on roughness of duct materials are listed in Table 1. A graphical presentation of the Colebrook equation developed by Moody [12] was used for decades to calculate friction factors. However today it is practical to solve the Colebrook equation with computers.

Local Loss Resistances: The pressure loss, Δp, through many other elements can be expressed as

$$\Delta p = C_o \frac{\rho U_o^2}{2} \tag{21}$$

where U_o is the average velocity at cross section o within the element, and C_o is a local loss coefficient. This equation is commonly used for inlets, outlets, duct contractions and expansions,

Table 1. Absolute roughness values for common duct materials

Duct Material	Roughness Category	Absolute Roughness, ε	
		mm	ft
Uncoated Carbon Steel, Clean. PVC Plastic Pipe. Aluminum.	Smooth	0.03	0.0001
Galvanized Steel, Longitudinal Seams, 1200 mm Joints. Galvanized Steel, Continuously Rolled, Spiral Seams, 3000 mm Joints. Galvanized Steel, Spiral Seam with 1, 2 and 3 Ribs, 3600 mm Joints.	Medium Smooth	0.09	0.0003
Galvanized Steel, Longitudinal Seams, 760 mm Joints.	Average	0.15	0.0005
Fibrous Glass Duct, Rigid. Fibrous Glass Duct Liner, Air Side With Facing Material.	Medium Rough	0.9	0.003
Fibrous Glass Duct Liner, Air Side Spray Coated. Flexible Duct, Metallic. Flexible Duct, All Types of Fabric and Wire. Concrete.	Rough	3.0	0.01

heating and cooling coils, dampers, bends and many filters. For a large number of these elements, values of C_o have been empirically determined and are tabulated frequently as functions of geometry in handbooks [10 - 12]. Manufacturers literature also contains some values of C_o. The value of K for these resistances is

$$K = \frac{C_o}{2\rho A_o^2}$$

where A_o is the area at cross section o.

Resistance of Junctions: Junctions may be either converging or diverging. For network representation, a node is located at cross section c of a junction. The pressure losses in the main section depends on the flow in it and on the flow in branch, and the loss in the branch depends on both the flow in it and in the main. The pressure loss in the branch is expressed as

$$p_b - p_c = C_{c,b} \frac{\rho_c U_c^2}{2} \tag{23}$$

and the pressure loss in the main is

$$p_s - p_c = C_{c,s} \frac{\rho_c U_c^2}{2} \tag{24}$$

where $C_{c,b}$ and $C_{c,s}$ are local loss coefficients, p, ρ, and U are pressure, density and average velocity at cross sections b, c and s as illustrated in figure 8. Local loss coefficients for many junctions have been experimentally evaluated, and they are tabulated as functions of geometry and various flow ratios in handbooks along with values of C_o. Converging junctions may have negative local loss coefficients. For example when the flow in the main is much greater than that in the branch, the gas in the branch is pulled by the greater flow similar to the operation of a carburetor.

Example: Fans were used for smoke control system experiments at the Plaza Hotel building in Washington DC [13]. For this example, the fire floor was exhausted and the stairwell was pressurized, and detailed information concerning this example is provided in section 0 of the main report. The exhaust fan from the corridor of the second floor has a capacity of about A 0.94 m^3/s (2000 cfm), and the stairwell pressurization fan is about 3.3 m^3/s (7000 cfm). Figure 7 shows these fan-duct systems, and the symbols used are those of CFAST/FAST. The fire floor exhaust fan is located between nodes 2 and 3, and the stairwell fan is located between nodes 6 and 7. Further there is a sign convention concerning the direction of normal flow through the fan. At the fan inlet, the node number is followed by a minus sign. At the outlet, the node number is preceded with a plus sign. For these examples the fans are specified at the constant volumetric flow rates listed above. For example, specifying a constant 0.94 m^3/s (2000 cfm) fan indicates that this fan will move 0.94 m^3/s (2000 cfm) of air regardless of the pressure across the fan. The constant flow rate fan is a useful approximation for many applications.

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